Simulating Jupiter's weather layer. Part I: Jet spin-up in a dry atmosphere

Supplementary Information

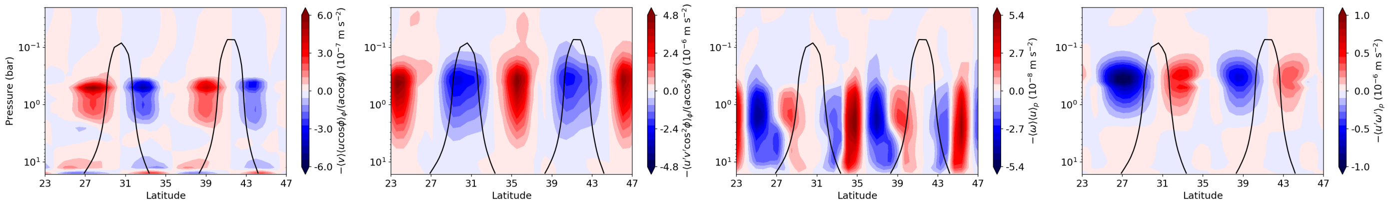
Roland M. B. Young, Peter L. Read, Yixiong Wang

Atmospheric, Oceanic and Planetary Physics, Department of Physics, University of Oxford, Oxford, UK.

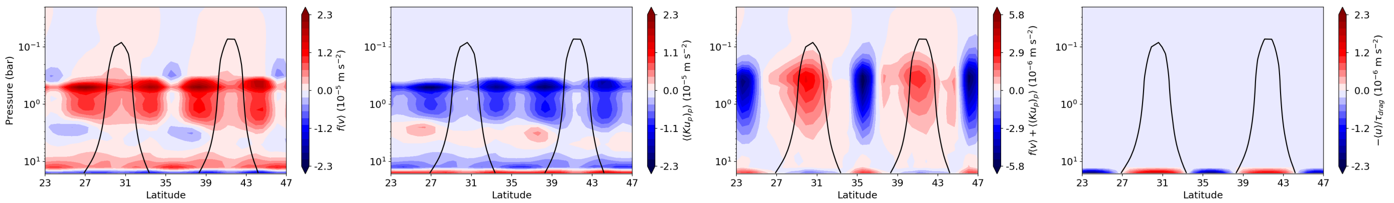
Contents

S1. Additional figures

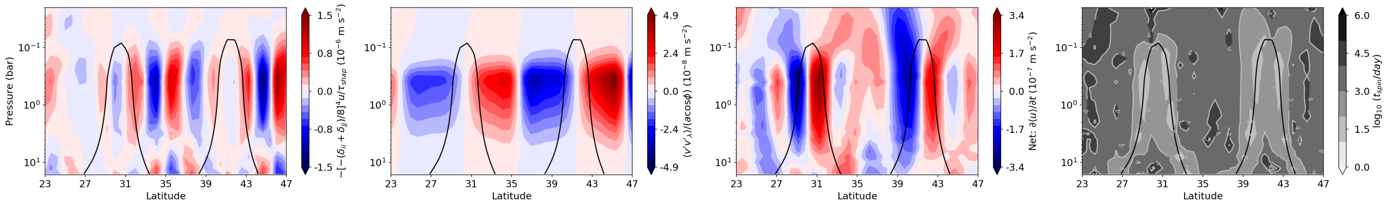
List of Figures



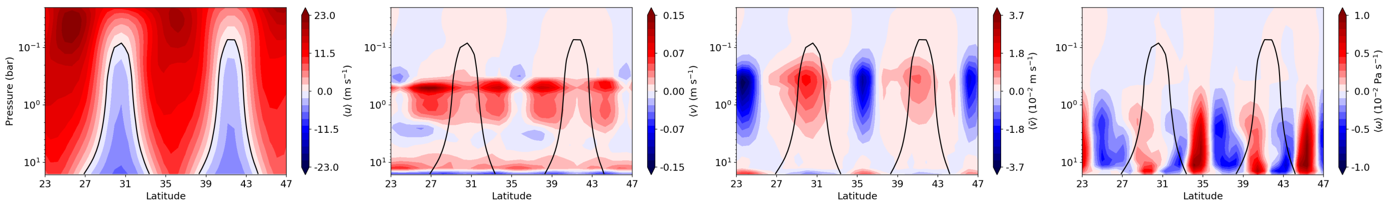
(a) Meridional mean AM flux convergence. (b) Meridional eddy AM flux convergence. (c) Vertical mean AM flux convergence. (d) Vertical eddy AM flux convergence.



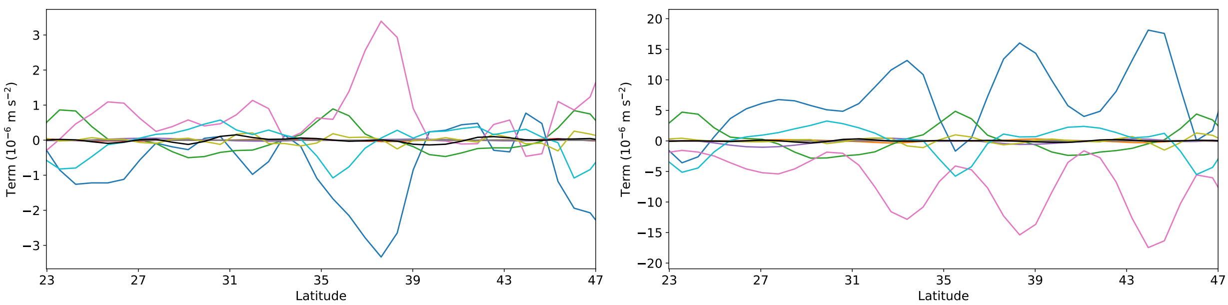
(e) Coriolis acceleration. (f) Vertical diffusion. (g) Adjusted Coriolis acceleration. (h) Bottom drag.



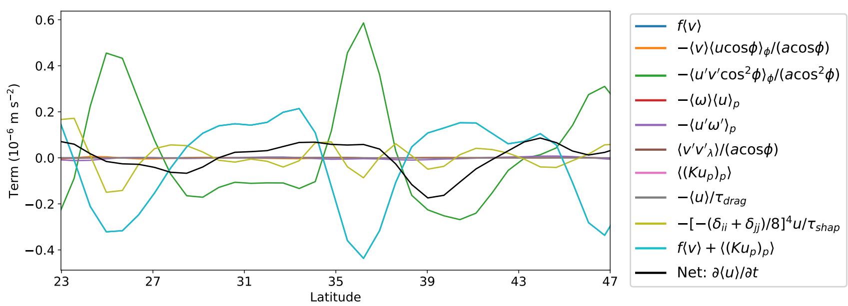
(i) Estimate of Shapiro filter. (j) $\langle \frac{v'}{a\cos\phi}\frac{\partial v'}{\partial\lambda}\rangle$ (k) Net $\partial\langle u\rangle/\partial t$. (l) Spin-down time.



(m) Zonal mean zonal velocity. (n) Zonal mean meridional velocity. (o) Adjusted zonal mean meridional velocity. (p) Zonal mean ω velocity (positive downwards).

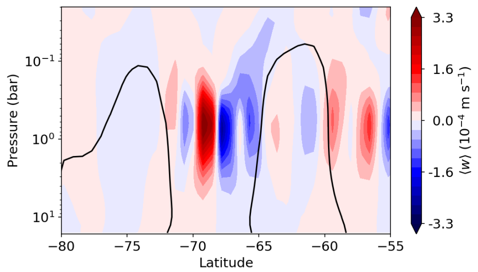


(q) Terms at 3.98 bar. (r) Terms at 0.40 bar.

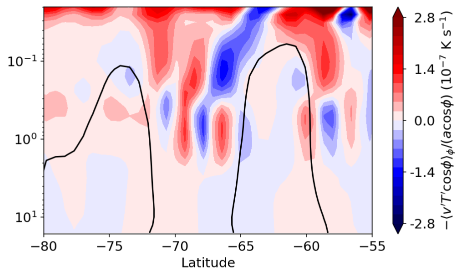


(s) Terms at 0.04 bar.

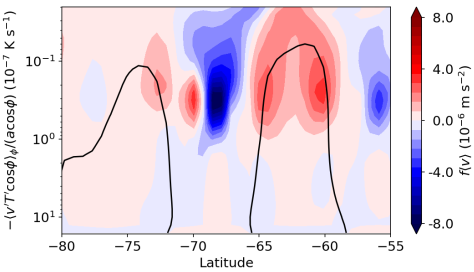
Figure S1: All the terms in the Eulerian mean equation (main text Eq. 9) during Run B in equilibrium at midlatitudes, averaged over 132900–133000 d. Also shown are the adjusted Coriolis acceleration, zonal mean velocity components, adjusted zonal mean meridional velocity (main text Eq. 10), spin-down time, and latitudinal profiles at three pressure levels. In (i), $\delta_{ii}u_{i,j} = u_{i+1,j} - 2u_{i,j} + u_{i-1,j}$, similarly for δ_{jj}, and $\{\cdots\}^4$ means to apply this operator four times (*Adcroft et al.*, 2018, Sect. 2.20).



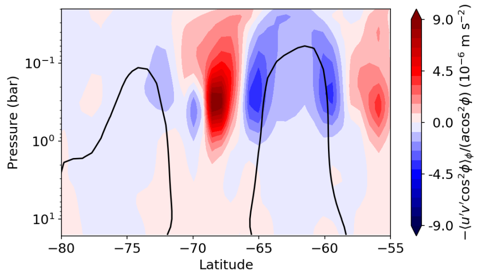
(a) Zonal mean vertical velocity (using $w = -\omega/(\rho g)$).



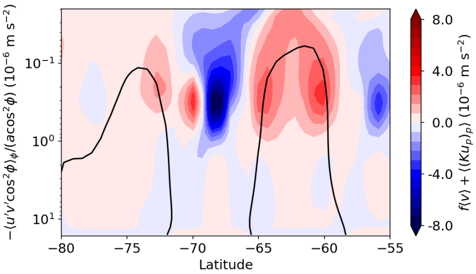
(b) Meridional eddy heat flux convergence.



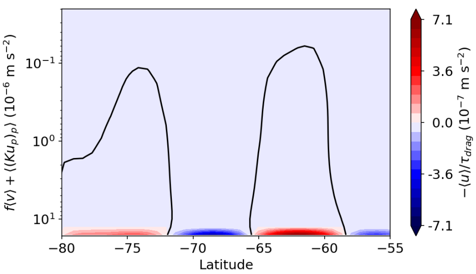
(c) Coriolis acceleration.



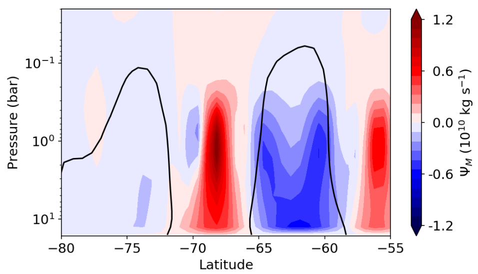
(d) Meridional eddy AM flux convergence.



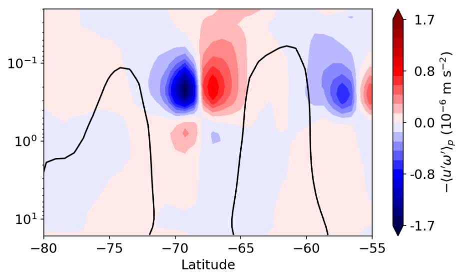
(e) Adjusted Coriolis acceleration.



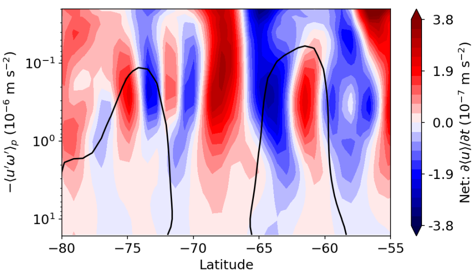
(f) Bottom drag.



(g) Mass streamfunction. Positive is clockwise.

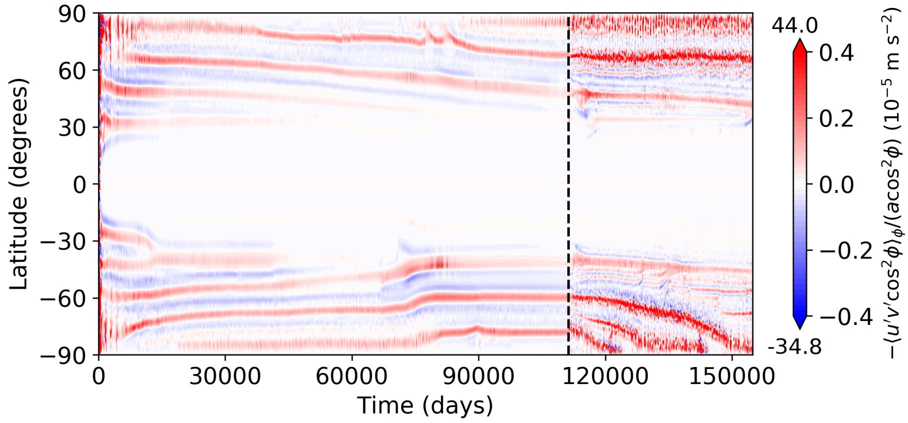


(h) Vertical eddy momentum flux convergence.

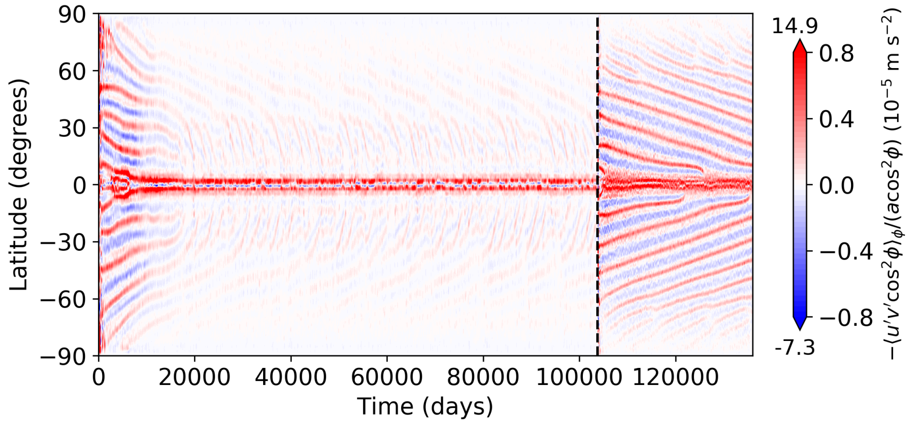


(i) Net forcing of zonal mean zonal velocity.

Figure S2: Meridional structure of high latitude jets for Run A in equilibrium. As Fig. 12 in the main text but for Run A between 80–55°S, averaged over days 154760–154860. Note that the Coriolis acceleration and the adjusted Coriolis acceleration are identical, so the mass streamfunction uses the normal zonal mean meridional velocity. Eastward jets peak around 69°S and 56°S.

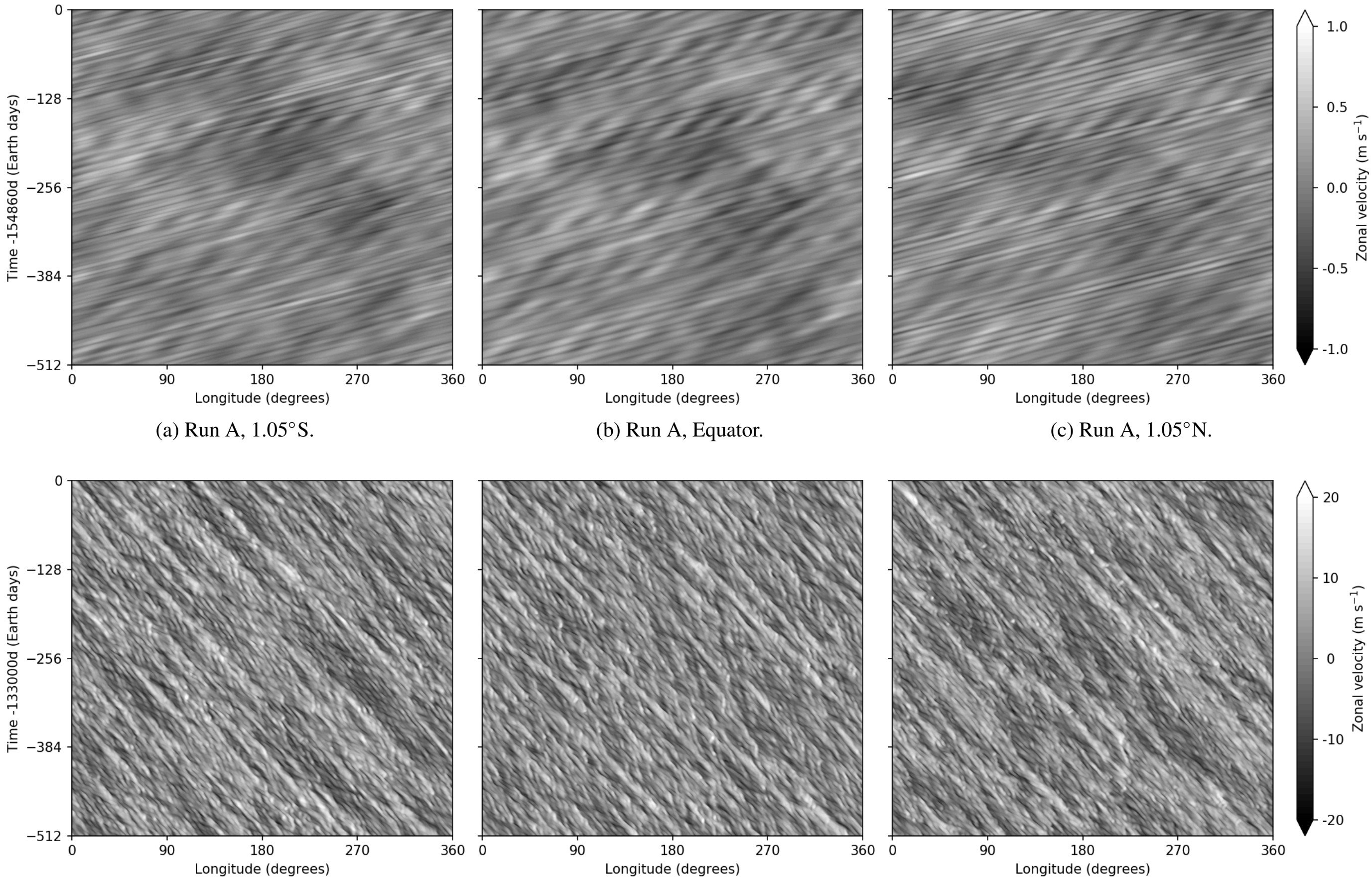


(a) Run A.



(b) Run B.

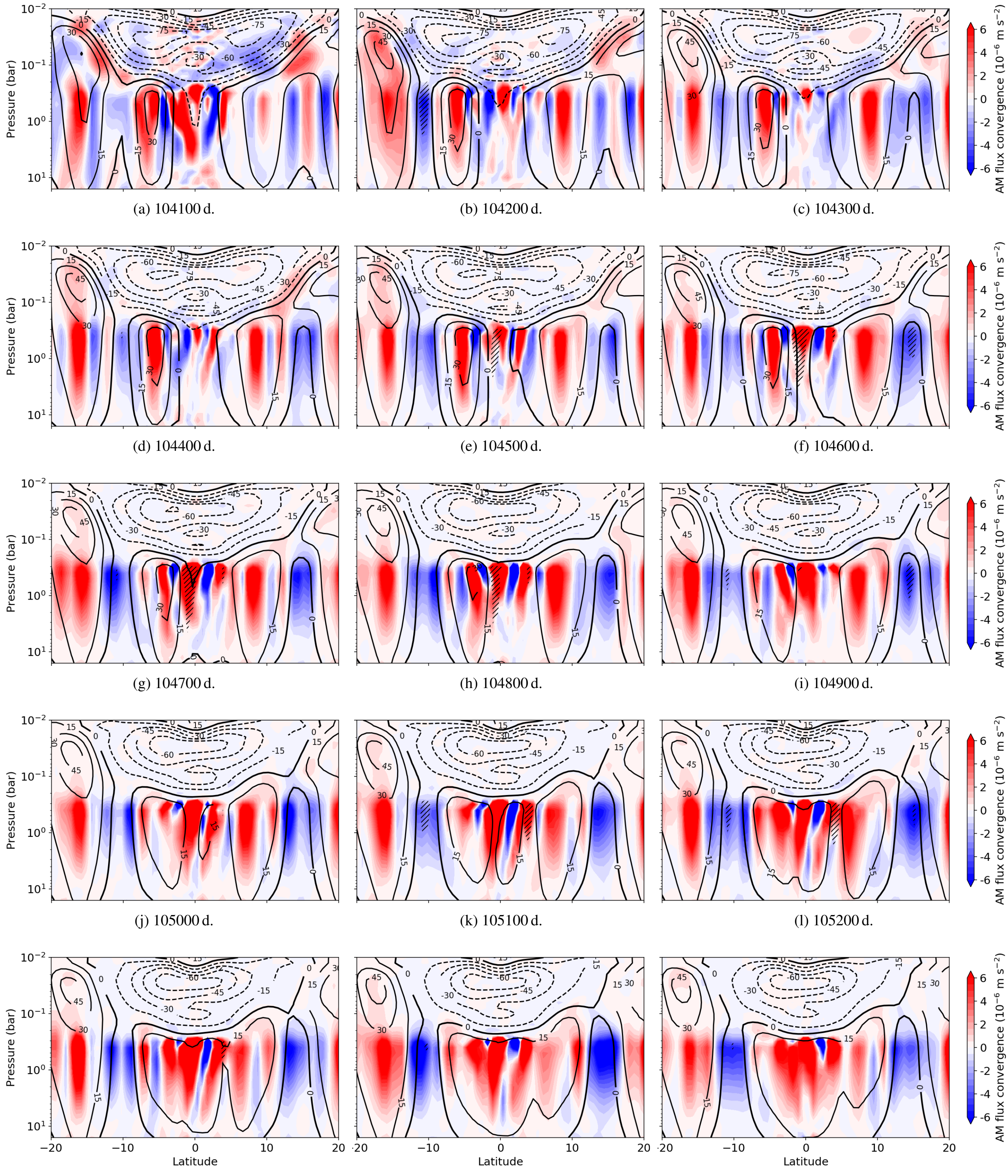
Figure S3: Like Fig. 13 in the main text, but showing Hovmöller diagrams for the meridional eddy momentum flux convergence at 1 bar over the whole of Runs A and B. The vertical dashed line shows when the resolution was doubled from L to M, and numbers at the ends of the colour bars show the minima and maxima.



(a) Run A, 1.05°S. (b) Run A, Equator. (c) Run A, 1.05°N.

(d) Run B, 1.05°S. (e) Run B, Equator. (f) Run B, 1.05°N.

Figure S4: As Fig. 19 in the main text but as well as showing Hovmöller plots of zonal velocity at the equator, it shows the same at two latitudes either side during Runs A and B in equilibrium at $p = 1$ bar. The three latitudes are the same as the horizontal lines in Fig. 18 in the main text.



(a) 104100 d. (b) 104200 d. (c) 104300 d.

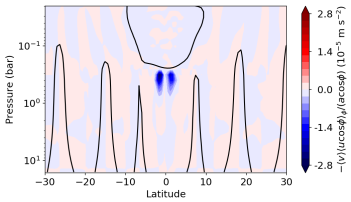
(d) 104400 d. (e) 104500 d. (f) 104600 d.

(g) 104700 d. (h) 104800 d. (i) 104900 d.

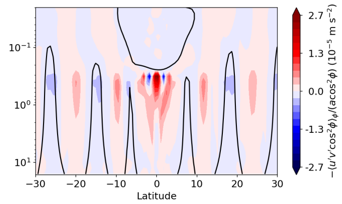
(j) 105000 d. (k) 105100 d. (l) 105200 d.

(m) 105300 d. (n) 105400 d. (o) 105500 d.

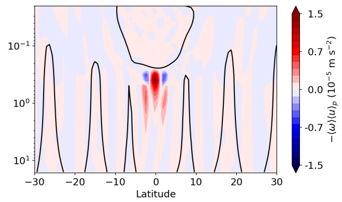
Figure S5: Complete sequence of latitude-pressure cross sections over 1500 d during the spin-up phase of Run B at resolution M. Frames are 100 d apart, beginning at 104100 d and ending at 105500 d. Colours show angular momentum flux convergence $-\partial(\langle u'v' \rangle \cos^2\phi)/(a\cos^2\phi\partial\phi)$, line contours show zonal mean zonal velocity $\langle u \rangle$ (eastward flow is solid, westward flow is dashed, and zero is thicker than the other contours), and hatched regions show where the flow is barotropically unstable, i.e. where $\partial^2\langle u \rangle/\partial y^2 > \beta$.



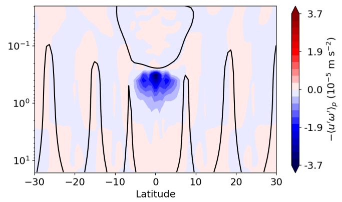
(a) Meridional mean AM flux convergence.



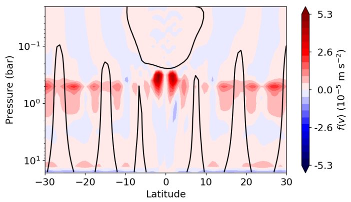
(b) Meridional eddy AM flux convergence.



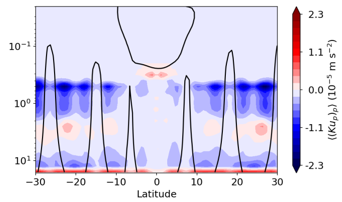
(c) Vertical mean AM flux convergence.



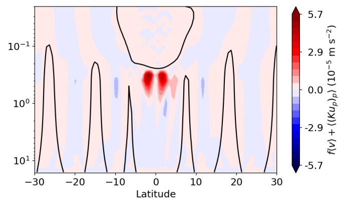
(d) Vertical eddy AM flux convergence.



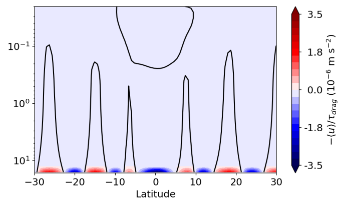
(e) Coriolis acceleration.



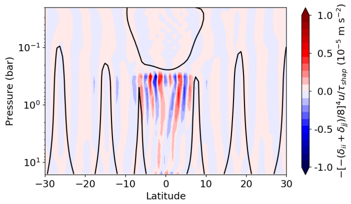
(f) Vertical diffusion.



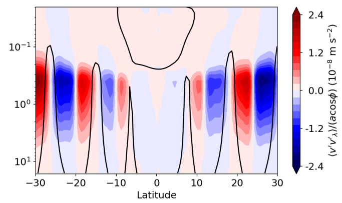
(g) Adjusted Coriolis acceleration.



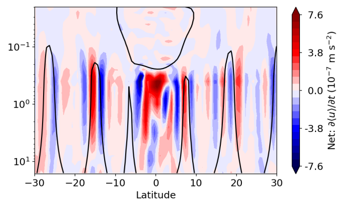
(h) Bottom drag.



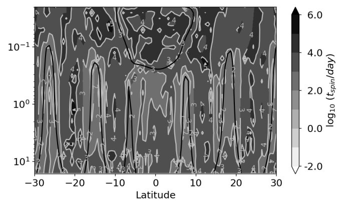
(i) Estimate of Shapiro filter.



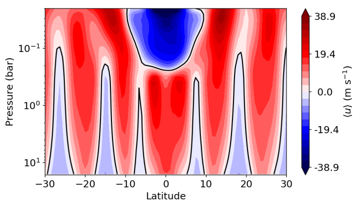
(j) $\langle \frac{v'}{a\cos\phi}\frac{\partial v'}{\partial \lambda} \rangle$



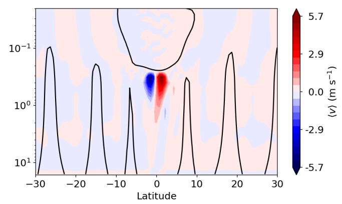
(k) Net $\partial\langle u\rangle/\partial t$.



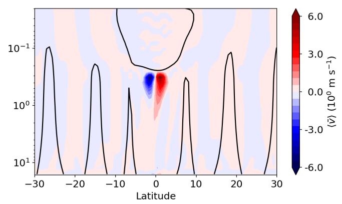
(l) Spin-down time.



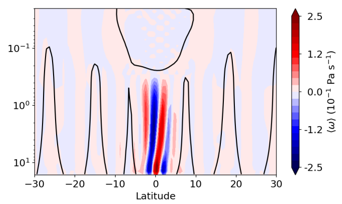
(m) Zonal mean zonal velocity.



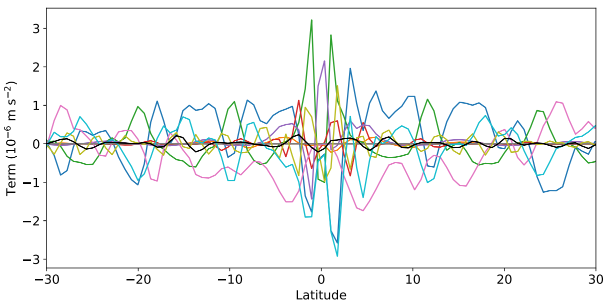
(n) Zonal mean meridional velocity.



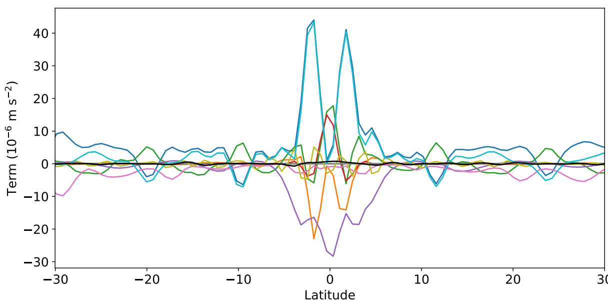
(o) Adjusted zonal mean meridional velocity.



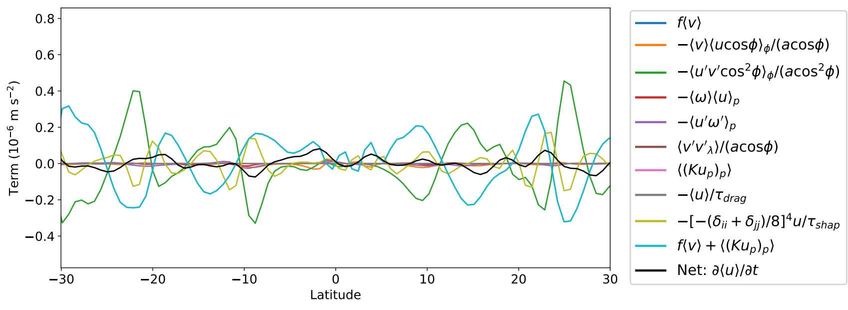
(p) Zonal mean ω velocity (positive downwards).



(q) Terms at 3.98 bar.

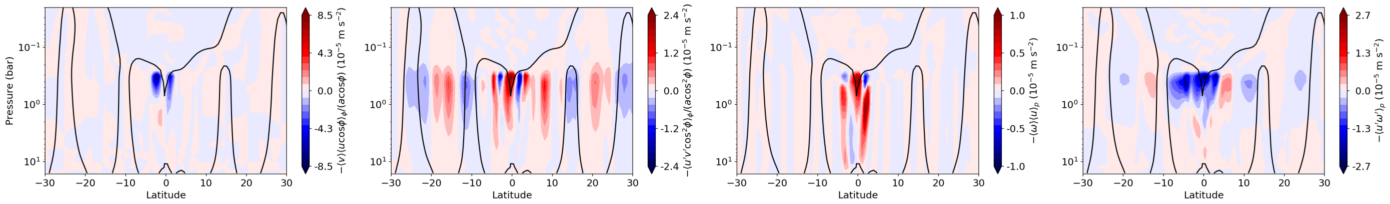


(r) Terms at 0.40 bar.

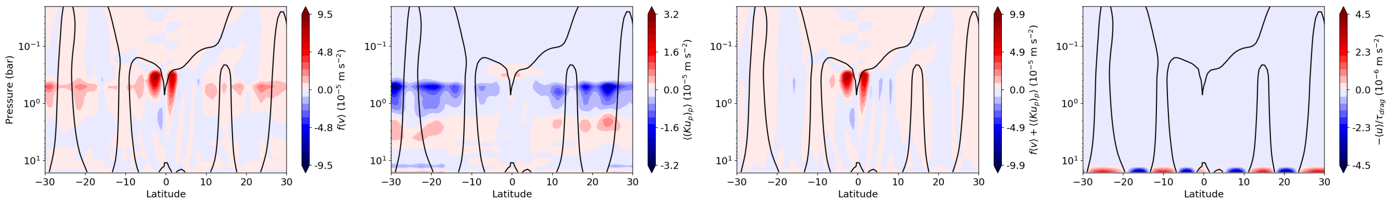


(s) Terms at 0.04 bar.

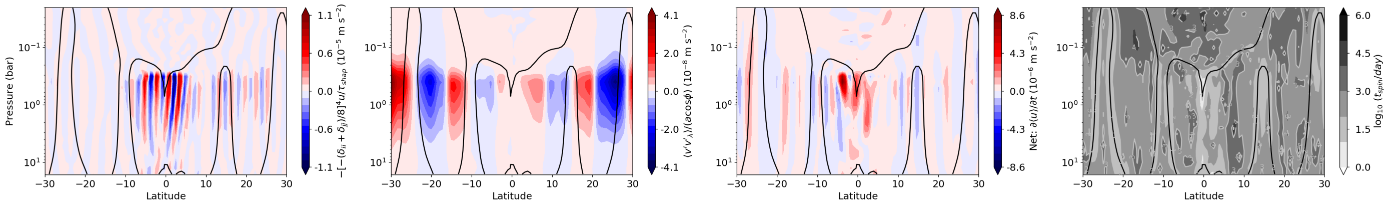
Figure S6: As Fig. S1 but for Run B in the equatorial region in equilibrium, averaged over 132900–133000 d.



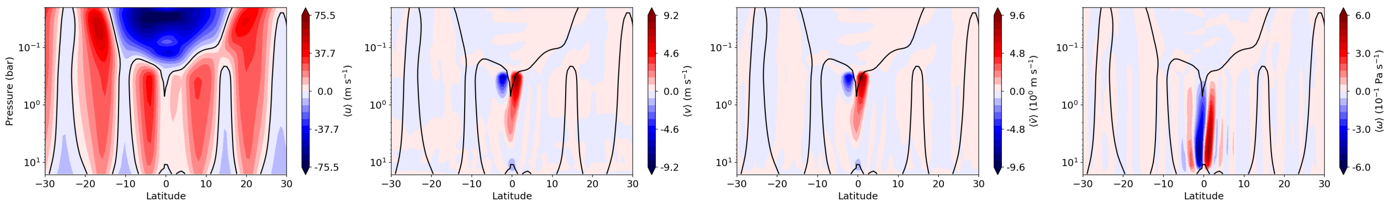
(a) Meridional mean AM flux convergence. (b) Meridional eddy AM flux convergence. (c) Vertical mean AM flux convergence. (d) Vertical eddy AM flux convergence.



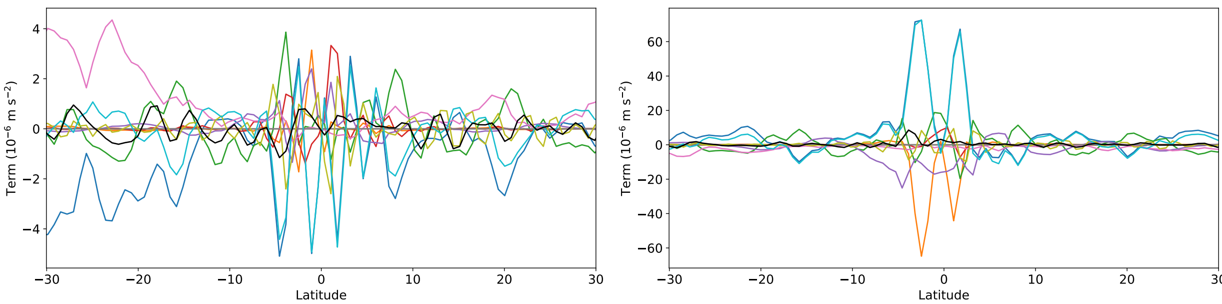
(e) Coriolis acceleration. (f) Vertical diffusion. (g) Adjusted Coriolis acceleration. (h) Bottom drag.



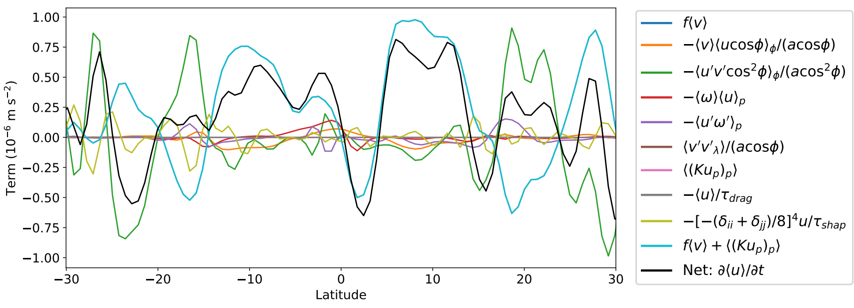
(i) Estimate of Shapiro filter. (j) $\langle \frac{v'}{a\cos\phi}\frac{\partial v'}{\partial\lambda}\rangle$ (k) Net $\partial\langle u\rangle/\partial t$. (l) Spin-down time.



(m) Zonal mean zonal velocity. (n) Zonal mean meridional velocity. (o) Adjusted zonal mean meridional velocity. (p) Zonal mean ω velocity (positive downwards).

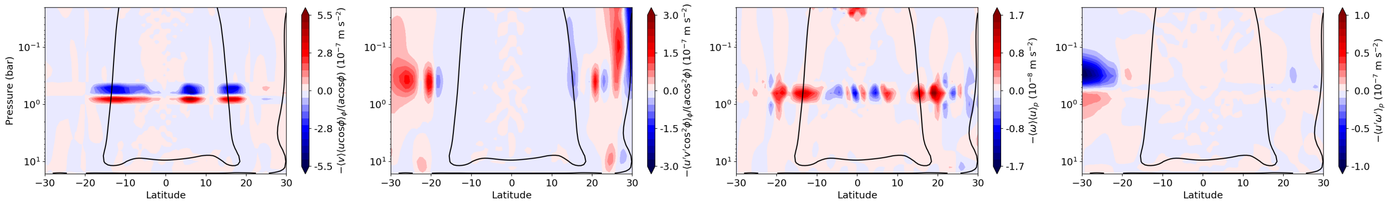


(q) Terms at 3.98 bar. (r) Terms at 0.40 bar.

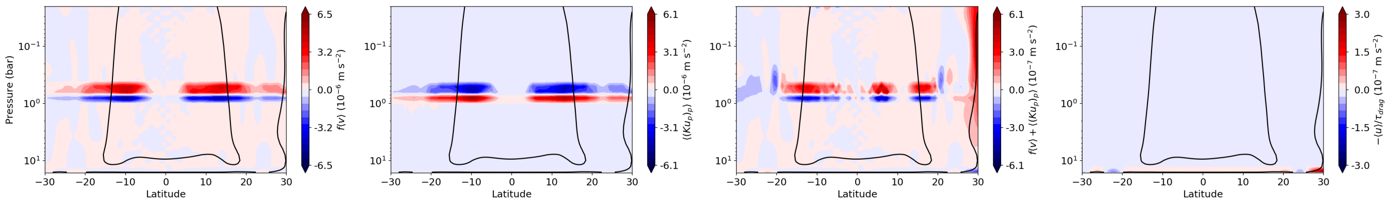


(s) Terms at 0.04 bar.

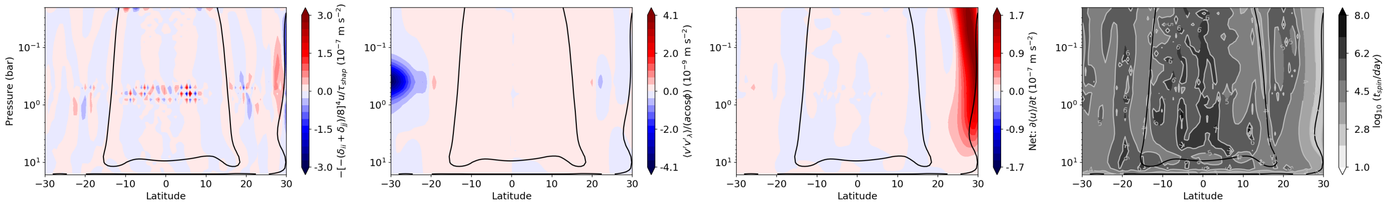
Figure S7: As Fig. S1 but for Run B in the equatorial region during spin-up, at 104700 d.



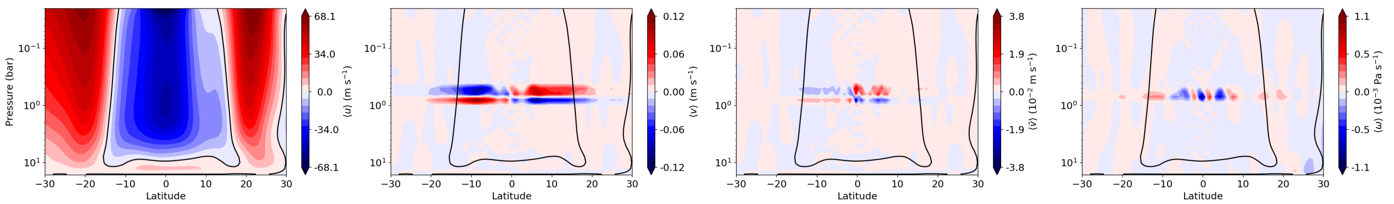
(a) Meridional mean AM flux convergence. (b) Meridional eddy AM flux convergence. (c) Vertical mean AM flux convergence. (d) Vertical eddy AM flux convergence.



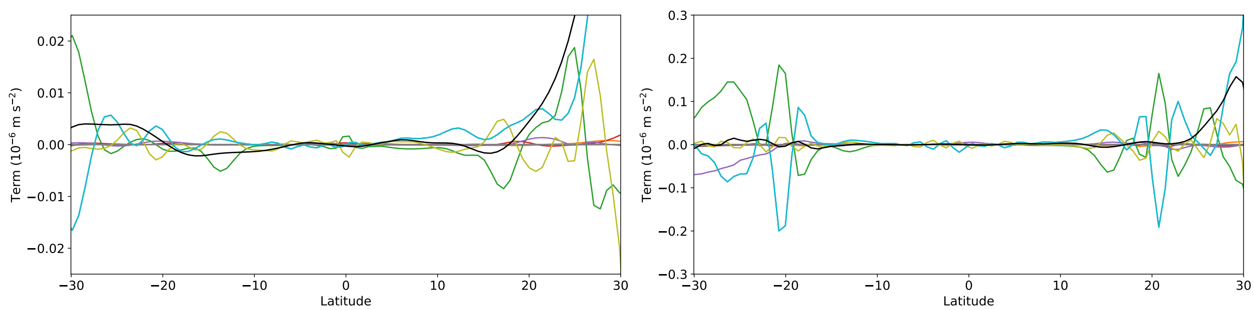
(e) Coriolis acceleration. (f) Vertical diffusion. (g) Adjusted Coriolis acceleration. (h) Bottom drag.



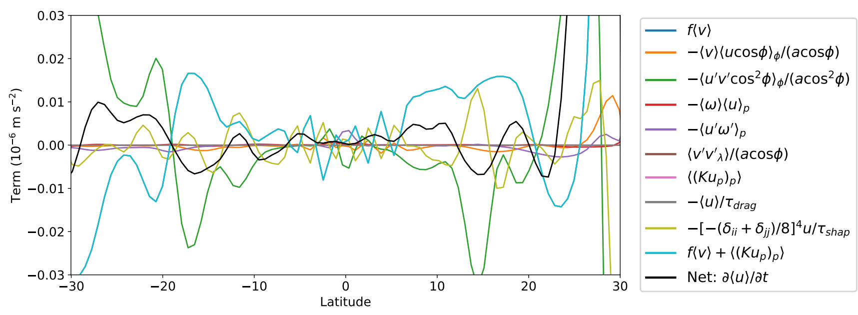
(i) Estimate of Shapiro filter. (j) $\langle \frac{v'}{a\cos\phi} \frac{\partial v'}{\partial \lambda} \rangle$ (k) Net $\partial\langle u \rangle/\partial t$. (l) Spin-down time.



(m) Zonal mean zonal velocity. (n) Zonal mean meridional velocity. (o) Adjusted zonal mean meridional velocity. (p) Zonal mean ω velocity.

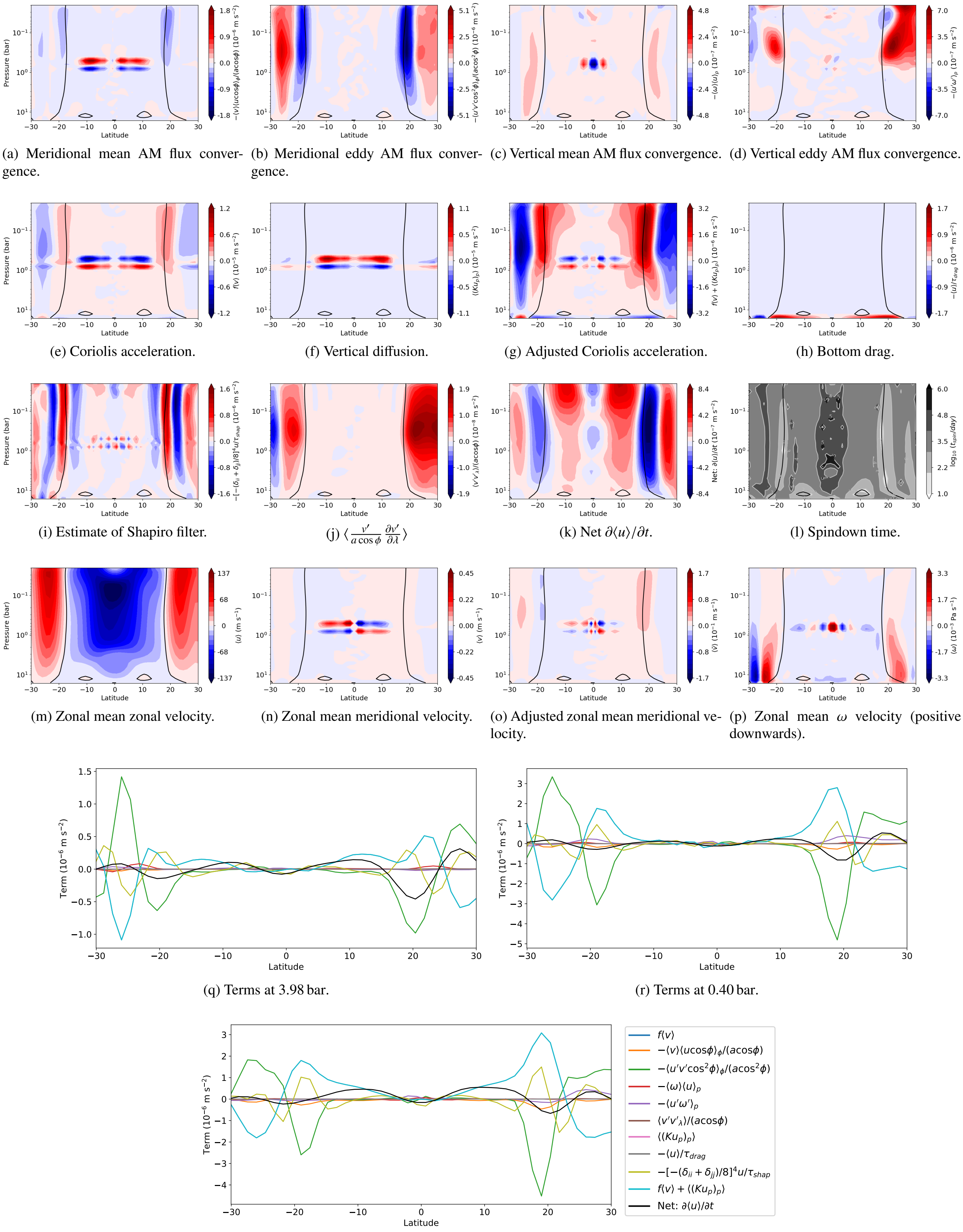


(q) Terms at 3.98 bar. (r) Terms at 0.40 bar.



(s) Terms at 0.04 bar.

Figure S8: As Fig. S1 but for Run A in the equatorial region in equilibrium, averaged over 153860–154860 d.



(a) Meridional mean AM flux convergence.

(b) Meridional eddy AM flux convergence.

(c) Vertical mean AM flux convergence.

(d) Vertical eddy AM flux convergence.

(e) Coriolis acceleration.

(f) Vertical diffusion.

(g) Adjusted Coriolis acceleration.

(h) Bottom drag.

(i) Estimate of Shapiro filter.

(j) $\langle \frac{v'}{a\cos\phi}\frac{\partial v'}{\partial\lambda}\rangle$

(k) Net $\partial\langle u\rangle/\partial t$.

(l) Spindown time.

(m) Zonal mean zonal velocity.

(n) Zonal mean meridional velocity.

(o) Adjusted zonal mean meridional velocity.

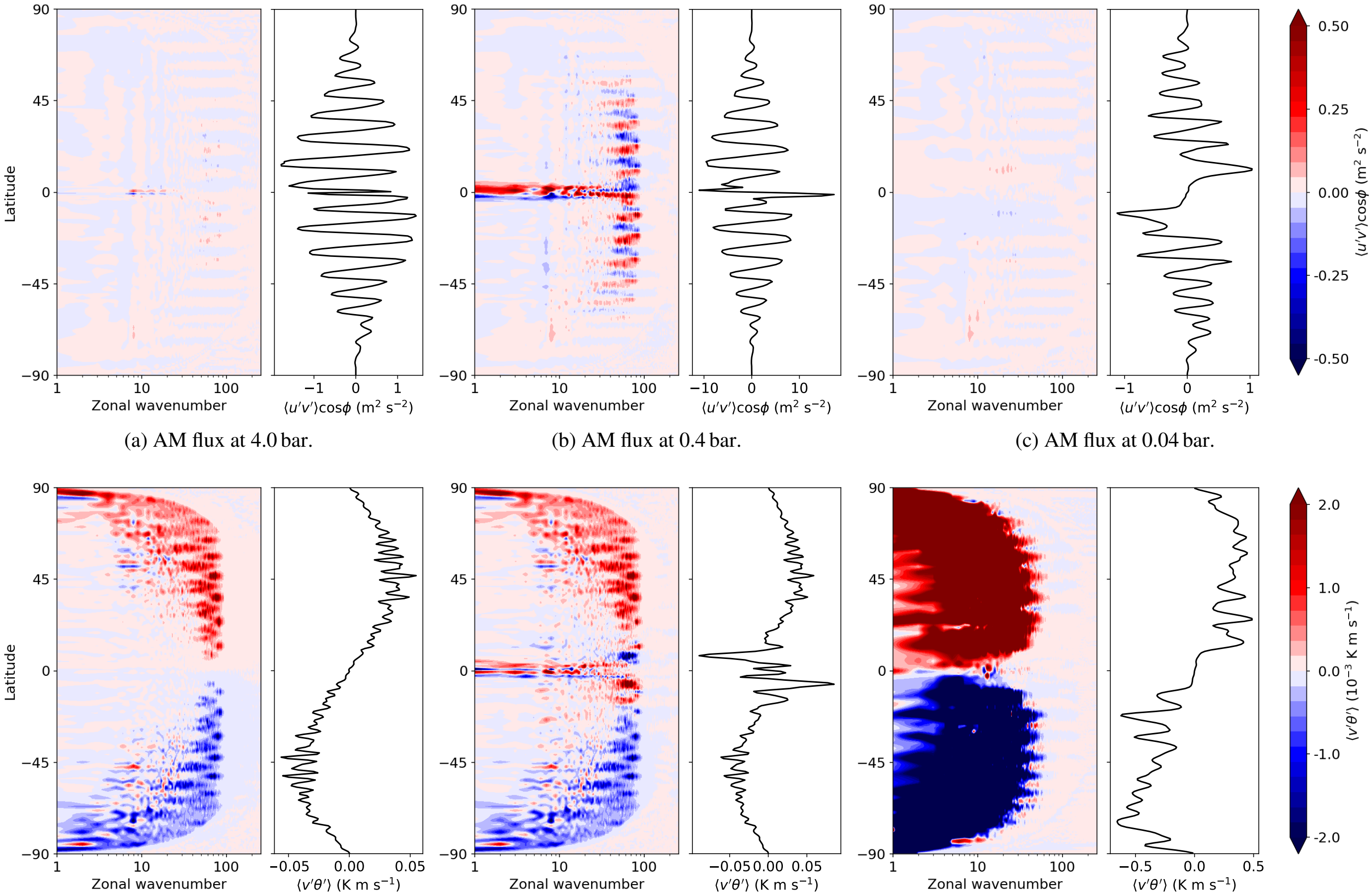
(p) Zonal mean ω velocity (positive downwards).

(q) Terms at 3.98 bar.

(r) Terms at 0.40 bar.

(s) Terms at 0.04 bar.

Figure S9: As Fig. S1 but for Run A in the equatorial region during spin-up, averaged over 1300–1500 d.



(a) AM flux at 4.0 bar.

(b) AM flux at 0.4 bar.

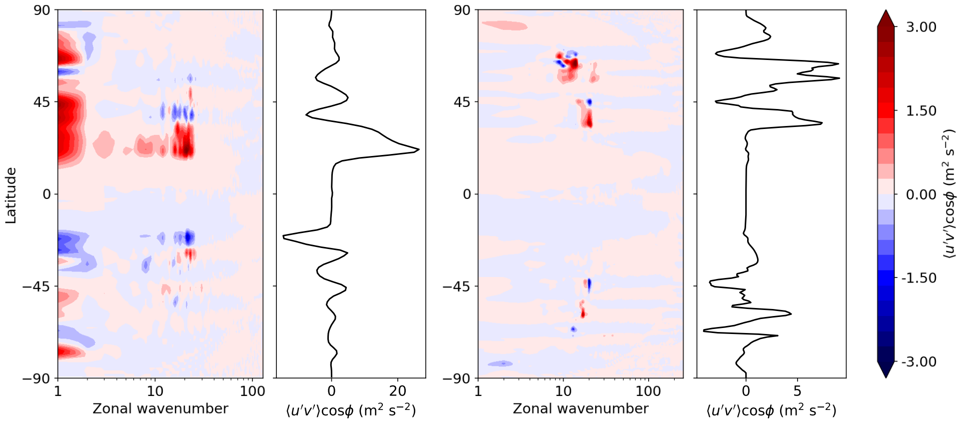
(c) AM flux at 0.04 bar.

(d) Heat flux at 4.0 bar.

(e) Heat flux at 0.4 bar.

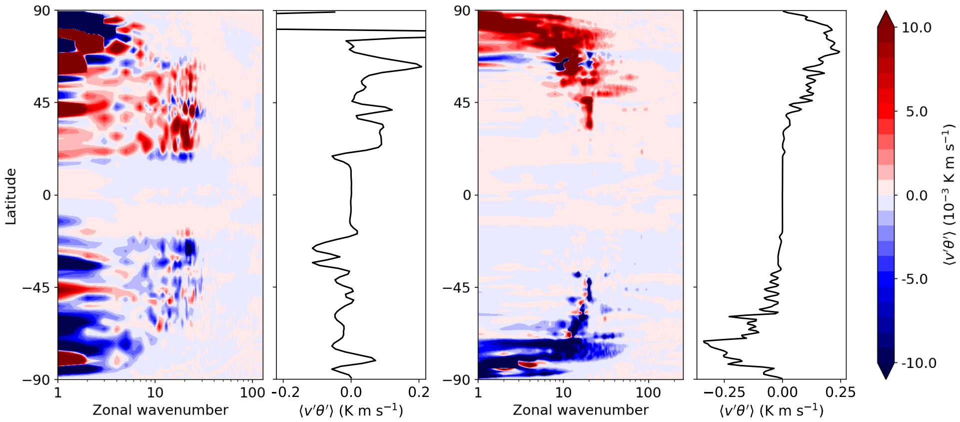
(f) Heat flux at 0.04 bar.

Figure S10: Co-spectra and associated latitudinal profiles for Run B in equilibrium (averaged over 132800–133000 d; note this is a slightly longer average than other figures at this time, to improve the signal to noise ratio, which is quite low for this diagnostic). Contours show angular momentum flux and heat flux co-spectra. Profiles show latitudinal profiles of AM flux $\langle u'v' \rangle \cos\phi$ and heat flux $\langle v'\theta' \rangle$, which equal the sum of the co-spectral coefficients at each latitude.



(a) Eddy AM flux during spin-up.

(b) Eddy AM flux in equilibrium.



(c) Eddy heat flux during spin-up.

(d) Eddy heat flux in equilibrium.

Figure S11: As Fig. S10 but showing eddy AM and heat flux co-spectra and profiles at 0.4 bar during Run A during spin-up (right at the start of the run, averaged over 1300–1500 d), and in equilibrium (averaged over the final 1000 d, between 153860–154860 d).

S2. Terms in the Eulerian mean equation calculated using the MITgcm's vector invariant discretisation

The Eulerian mean equation is (e.g. *Andrews et al.*, 1983, Eq. 2.5)

$$\frac{\partial\langle u\rangle}{\partial t} = -\frac{\partial(\langle u\rangle\cos\phi))}{a\cos\phi\,\partial\phi}\langle v\rangle - \frac{\partial(\langle u'v'\rangle\cos^2\phi))}{a\cos\phi^2\,\partial\phi} - \frac{\partial\langle u\rangle}{\partial p}\langle\omega\rangle - \frac{\partial\langle u'\omega'\rangle}{\partial p} + f\langle v\rangle \tag{S1}$$

where $\langle\cdot\rangle$ denotes a zonal mean and $'$ the deviation therefrom, e.g. $u(\lambda,\phi,p) = \langle u\rangle(\phi,p) + u'(\lambda,\phi,p)$. In its vector invariant form, the various forcing terms on the right hand side of the MITgcm zonal momentum equation (*Adcroft et al.*, 2018, Sect. 2.12) are (*ibid.*, Sect. 2.15):

$$G_u = G_u^{fv} + G_u^{\zeta_3 v} + G_u^{\zeta_2 w} + G_u^{\partial_x B} + G_u^{\partial_z \tau^x} + G_u^{h-dissip} + G_u^{v-dissip}$$

The first three of these terms represent the terms on the right hand side of Eq. S1 before the zonal mean is taken. In the Jupiter model $G_u^{h-dissip} = G_u^{v-dissip} = 0$ as all dissipation parameters are zero, and $\langle G_u^{\partial_x B}\rangle = 0$ as this term is a pure longitudinal derivative (*ibid.*, Sect. 2.15.5) that sums to zero when the zonal mean is taken. $G_u^{\partial_z\tau^x}$ is the surface stress and so is also not relevant here. The other terms introduced into the Jupiter model discussed in the main text are not discussed here.

There is a delicate balance between these terms. If they are calculated using some naïve estimate of horizontal derivatives, instead of expressions derived from the same (vector invariant) equations that the model uses, then the net acceleration is typically of the same order of magnitude as individual terms, but such a net acceleration is not reflected in how the zonal mean zonal velocity actually changes over time.

The longitudinal index is i, the latitudinal index is j, and the vertical index is k (counting from the bottom of the atmosphere). On the longitude-latitude Arakawa C grid (*ibid.*, Sect. 2.11.4) all the Δy are the same: $\Delta y^f = \Delta y^u = \Delta y^c = \Delta y^g = \Delta y$, the Δx at the same latitude are the same, and independent of longitude: $\Delta x_j^g = \Delta x_j^v$ and $\Delta x_j^c = \Delta x_j^f$, and the area elements spanning the same latitudes are the same, and independent of longitude: $A_j^c = A_j^w$ and $A_j^\zeta = A_j^s$.

Linear Coriolis term. The first term on the right hand side of Eq. S1 is (*ibid.*, Sect. 2.15.3)

$$G_u^{fv} = \frac{1}{\Delta x^c}\overline{f}^j\,\overline{\overline{\Delta x^g v}^j}^i$$

where we use the following notation (*ibid.*, Sect. 2.1):

$$\overline{\Phi}^i = \frac{\Phi_{i-1/2,j,k} + \Phi_{i+1/2,j,k}}{2} \qquad \overline{\Phi}^j = \frac{\Phi_{i,j-1/2,k} + \Phi_{i,j+1/2,k}}{2} \qquad \overline{\Phi}^k = \frac{\Phi_{i,j,k-1/2} + \Phi_{i,j,k+1/2}}{2}$$

We omit the h terms throughout as there is no bathymetry and so they all equal one. The full discretised form of this term is

$$G_u^{fv}\Big|_{ij} = \frac{1}{\Delta x_j^c}\frac{f_j^v + f_{j+1}^v}{2}\left[\frac{\Delta x_j^g(v_{ij} + v_{i-1j}) + \Delta x_{j+1}^g(v_{ij+1} + v_{i-1j+1})}{4}\right]$$

Taking the zonal mean of this equation we get

$$f\langle v\rangle = \langle G_u^{fv}\rangle_j = \frac{f_j^v + f_{j+1}^v}{2}\,\frac{\langle v\rangle_j\Delta x_j^g + \langle v\rangle_{j+1}\Delta x_{j+1}^g}{2\Delta x_j^c} \tag{S2}$$

where $\langle v\rangle_j$ is the zonal mean of v along a longitude circle, and $v_{ij} = \langle v\rangle_j + v'_{ij}$ splits the velocity into its zonal mean and eddy parts. In this term all the eddy terms sum to zero.

Nonlinear Coriolis term. The second term on the right hand side of Eq. S1 is (*ibid.*, Sect. 2.15.3)

$$G_u^{\zeta_3 v} = \frac{1}{\Delta x^c}\overline{\zeta_3}^j\,\overline{\overline{\Delta x^g v}^j}^i$$

where the relative vorticity is (*ibid.*, Sect. 2.15.1)

$$\zeta_3 = \frac{\delta_i[\Delta y^c v] - \delta_j[\Delta x^c u]}{A^\zeta}$$

where we use the following notation (*ibid.*, Sect. 2.1):

$$\delta_i\Phi = \Phi_{i+1/2,j,k} - \Phi_{i-1/2,j,k} \qquad \delta_j\Phi = \Phi_{i,j+1/2,k} - \Phi_{i,j-1/2,k} \qquad \delta_k\Phi = \Phi_{i,j,k+1/2} - \Phi_{i,j,k-1/2}$$

The full form of this term is

$$
\begin{aligned}
G_u^{\zeta_3 v}\Big|_{ij} &= \frac{1}{2}\left[\frac{\Delta y(v_{ij}-v_{i-1j})-(\Delta x^c_j u_{ij}-\Delta x^c_{j-1}u_{ij-1})}{A^\zeta_j}+\frac{\Delta y(v_{ij+1}-v_{i-1j+1})-(\Delta x^c_{j+1}u_{ij+1}-\Delta x^c_j u_{ij})}{A^\zeta_{j+1}}\right]\\
&\quad\times\left[\frac{\Delta x^g_j(v_{i-1j}+v_{ij})+\Delta x^g_{j+1}(v_{i-1j+1}+v_{ij+1})}{4\Delta x^c_j}\right]
\end{aligned}
$$

Splitting into zonal and eddy terms and taking the zonal mean, we end up with three terms:

$$
-\langle v\rangle\frac{\partial\langle u\rangle}{\partial y}\bigg|_j = -\frac{1}{4}\frac{\Delta x^g_j\langle v\rangle_j+\Delta x^g_{j+1}\langle v\rangle_{j+1}}{\Delta x^c_j}\left[\frac{\Delta x^c_j\langle u\rangle_j-\Delta x^c_{j-1}\langle u\rangle_{j-1}}{A^\zeta_j}+\frac{\Delta x^c_{j+1}\langle u\rangle_{j+1}-\Delta x^c_j\langle u\rangle_j}{A^\zeta_{j+1}}\right] \tag{S3}
$$

$$
-\left\langle v'\frac{\partial u'}{\partial y}\right\rangle_j = -\frac{1}{N_{lon}}\sum_{i=1}^{N_{lon}}\frac{1}{2}\left[\frac{\Delta x^c_j u'_{ij}-\Delta x^c_{j-1}u'_{ij-1}}{A^\zeta_j}+\frac{\Delta x^c_{j+1}u'_{ij+1}-\Delta x^c_j u'_{ij}}{A^\zeta_{j+1}}\right]\left[\frac{\Delta x^g_j(v'_{i-1j}+v'_{ij})+\Delta x^g_{j+1}(v'_{i-1j+1}+v'_{ij+1})}{4\Delta x^c_j}\right] \tag{S4}
$$

$$
\left\langle v'\frac{\partial v'}{\partial x}\right\rangle_j = \frac{\Delta y}{N_{lon}}\sum_{i=1}^{N_{lon}}\frac{1}{2}\left[\frac{v'_{ij}-v'_{i-1j}}{A^\zeta_j}+\frac{v'_{ij+1}-v'_{i-1j+1}}{A^\zeta_{j+1}}\right]\left[\frac{\Delta x^g_j(v'_{i-1j}+v'_{ij})+\Delta x^g_{j+1}(v'_{i-1j+1}+v'_{ij+1})}{4\Delta x^c_j}\right] \tag{S5}
$$

where N_{lon} is the number of longitudinal grid points, wrapping around in longitude where necessary. The term $\langle v\rangle\partial\langle v\rangle/\partial x$ is identically zero. In the continuous equations Eq. S5 also equals zero, but because of the discretisation it is actually nonzero (but very small). It has a very similar appearance to the horizontal metric term derived from the flux form equations (metric terms are explicitly excluded from the vector invariant equations).

Shear term. The third term on the right hand side of Eq. S1 is (*ibid.*, Sect. 2.15.4)

$$
G_u^{\zeta_2 w} = \frac{1}{A^w\Delta r^f}\overline{\overline{A^c w}^i\,\delta_k u}^k
$$

Where we have omitted the non-hydrostatic term as we do not use it. The full form of this term in pressure coordinates is

$$
G_u^{\zeta_2 w}\Big|_{ik} = -\frac{(\omega_{i-1k}+\omega_{ik})(u_{ik}-u_{ik-1})+(\omega_{i-1k+1}+\omega_{ik+1})(u_{ik+1}-u_{ik})}{4\Delta p^f_k}
$$

where $\Delta p^f_k = p^f_k - p^f_{k+1}$ is the pressure difference between faces k and $k+1$, and is defined to be positive. Splitting into zonal and eddy terms and computing the zonal mean, we end up with two terms:

$$
-\langle\omega\rangle\frac{\partial\langle u\rangle}{\partial p}\bigg|_k = -\frac{\langle\omega\rangle_k(\langle u\rangle_k-\langle u\rangle_{k-1})+\langle\omega\rangle_{k+1}(\langle u\rangle_{k+1}-\langle u\rangle_k)}{2\Delta p^f_k} \tag{S6}
$$

$$
-\left\langle\omega'\frac{\partial u'}{\partial p}\right\rangle_k = -\frac{1}{4N_{lon}\Delta p^f_k}\sum_{i=1}^{N_{lon}}\left[(\omega'_{i-1k}+\omega'_{ik})(u'_{ik}-u'_{ik-1})+(\omega'_{i-1k+1}+\omega'_{ik+1})(u'_{ik+1}-u'_{ik})\right] \tag{S7}
$$

Eddy fluxes. At this point we have three of the five terms in the Eulerian mean equation, the first, third, and fifth terms, but we do not yet have the eddy fluxes. We require $-\partial\langle u'v'\rangle/\partial y$ but have $-\langle v'\partial u'/\partial y\rangle$, and $-\partial\langle u'\omega'\rangle/\partial p$ but have $-\langle\omega'\partial u'/\partial p\rangle$. These are related by

$$
-\frac{\partial\langle u'v'\rangle}{\partial y} = -\left\langle v'\frac{\partial u'}{\partial y}\right\rangle - \left\langle u'\frac{\partial v'}{\partial y}\right\rangle \quad\text{and}\quad -\frac{\partial\langle u'\omega'\rangle}{\partial p} = -\left\langle\omega'\frac{\partial u'}{\partial p}\right\rangle - \left\langle u'\frac{\partial\omega'}{\partial p}\right\rangle
$$

By exploiting the symmetry of these two expressions, we can draw stencils for the terms we do know and use the symmetry of these stencils about a reflection in $i-j$ (or $i-k$ for the vertical term) to write down the other terms immediately, without doing any derivations:

$$
-\left\langle u'\frac{\partial v'}{\partial y}\right\rangle_j = -\frac{1}{4N_{lon}A^c_j}\sum_{i=1}^{N_{lon}}\left[(\Delta x^g_{j+1}v'_{i-1j+1}-\Delta x^g_j v'_{i-1j})(u'_{i-1j}+u'_{ij})+(\Delta x^g_{j+1}v'_{ij+1}-\Delta x^g_j v'_{ij})(u'_{ij}+u'_{ij+1})\right] \tag{S8}
$$

$$
-\left\langle u'\frac{\partial\omega'}{\partial p}\right\rangle_k = -\frac{1}{4N_{lon}\Delta p^f_k}\sum_{i=1}^{N_{lon}}\left[(u'_{i-1k}+u'_{ik})(\omega'_{i-1k+1}-\omega'_{i-1k})+(u'_{ik}+u'_{i+1k})(\omega'_{ik+1}-\omega'_{ik})\right] \tag{S9}
$$

If these two terms sum to zero then we can add both to $\langle G_u \rangle$ and not change its sum, which then allows us to express $\langle G_u \rangle$ in terms of the standard eddy fluxes $-\partial\langle u'v' \rangle/\partial y$ and $-\partial\langle u'\omega' \rangle/\partial p$. The sum of these two terms is

$$-\left\langle u' \frac{\partial v'}{\partial y} \right\rangle - \left\langle u' \frac{\partial \omega'}{\partial p} \right\rangle = -\left\langle u' \left(\frac{\partial v'}{\partial y} + \frac{\partial \omega'}{\partial p} \right) \right\rangle = -\left\langle u' \left(\frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} - \frac{\partial \langle v \rangle}{\partial y} - \frac{\partial \langle \omega \rangle}{\partial p} \right) \right\rangle$$

By continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

taking the zonal mean,

$$\frac{\partial \langle v \rangle}{\partial y} + \frac{\partial \langle \omega \rangle}{\partial p} = 0$$

So the latter two terms sum to zero, and the first and second can be rewritten as

$$\left\langle u' \frac{\partial u}{\partial x} \right\rangle = \left\langle u \frac{\partial u}{\partial x} - \langle u \rangle \frac{\partial u}{\partial x} \right\rangle = \frac{1}{2} \left\langle \frac{\partial (u^2)}{\partial x} \right\rangle - \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} = 0$$

So, in summary, we may rewrite the Eulerian mean equation Eq. S1 as

$$\begin{aligned}
\frac{\partial \langle u \rangle}{\partial t} &= (\text{Eq. S3}) && \text{Eq. S1 term 1 -- Meridional mean momentum flux convergence} \\
&+ (\text{Eq. S4 + Eq. S8}) && \text{Eq. S1 term 2 -- Meridional eddy momentum flux convergence} \\
&+ (\text{Eq. S6}) && \text{Eq. S1 term 3 -- Vertical mean momentum flux convergence} \\
&+ (\text{Eq. S7 + Eq. S9}) && \text{Eq. S1 term 4 -- Vertical eddy momentum flux convergence} \\
&+ (\text{Eq. S2}) && \text{Eq. S1 term 5 -- Coriolis term} \\
&+ (\text{Eq. S5}) && \text{Discretised version of } \langle v' \partial v' / \partial x \rangle
\end{aligned} \tag{S10}$$

References

Adcroft, A., J.-M. Campin, E. Doddridge, S. Dutkiewicz, C. Evangelinos, D. Ferreira, M. Follows, G. Forget, B. Fox-Kemper, P. Heimbach, C. Hill, E. Hill, H. Hill, O. Jahn, J. Klymak, M. Losch, J. Marshall, G. Maze, M. Mazloff, D. Menemenlis, A. Molod, and J. Scott (2018), MITgcm Documentation, Release checkpoint67a-12-gbf23121, 19 March 2018.

Andrews, D. G., J. D. Mahlman, and R. W. Sinclair (1983), Eliassen-Palm Diagnostics of Wave-Mean Flow Interaction in the GFDL “SKYHI” General Circulation Model, *J. Atmos. Sci.*, 40:2768–2784, doi:10.1175/1520-0469(1983)040<2768:ETWATM>2.0.CO;2.